[0132] v_z velocity magnitude in the axial dimension

[0133] θ wall adhesion contact angle

[0134] Channel Geometry

[0135] H channel gap

[0136] W channel width

[0137] L channel length

[0138] D_H channel hydraulic diameter

[0139] Other

[0140] z axial coordinate (z=0⇒ initial fill line)

[0141] β angle of incline (β=0⇒ vertical orientation)

[0142] θ gravitational constant

[0143] Drain Rate

[0144] Drain rate is based on a differential equation describing the change in total gravitational, viscous loss (shear flow), and capillary forces as a function of change in fluid level as it drains from the channel. The force balance involving all three components (gravitational, viscous loss, and capillary) is exactly solvable but numerically difficult to compute in practice. What follows below is an approximate solution to this problem with capillary force neglected, again with details of the derivation located in the appendix.

[0145] When the fluid level has dropped the equivalent to several hydraulic diameters below in the initial liquid fill line, the average velocity magnitude of the draining fluid is given by the following expression:

$$u_{drain} \sim u_o = \frac{\rho g D_H^2}{16\mu}\cos\beta$$

[0146] Initial Film Thickness

[0147] The initial average film thickness is based on a model by Landau and Levich (1942) and Deryagin (1943, 1945) where they studied the residual liquid layer remaining on a flat surface drawn at constant rate from a quiescent bath. The model is stated mathematically as follows:

$$\delta_o = \left(\frac{\mu u_o}{\rho g}\right)^{1/2} f(\xi)$$

[0148] where $\xi=\mu u_o/\sigma$ and f represents a function of this dimensionless variable ξ, namely

$$f(\xi) = \begin{cases} 0.93\xi^{1/6} \text{ for } \xi << 1 \\ 1 \text{ for } \xi >> 1 \end{cases}$$

[0149] This model is reported to have been experimentally validated by Deryagin and Titiyevskaya (1945). Although it appears to provide reasonable results, one drawback to this expression is that there is no explicit dependence on length of the plate. Intuitively one expects length to play a role since it determines the overall force of gravity and surface shear stress on the liquid layer. An alternative expression for initial average film thickness is given by Levich (1962):

$$\delta_o = \left(\frac{\sigma}{\rho g}\right)\left(\frac{F_\tau}{\sigma}\right)^2$$

[0150] where F_τ represents the total wall shear stress integrated over the entire surface area of the liquid-solid interface. Unfortunately, sample calculations showed that this expression yields unrealistically small values for initial film thickness. Two expressions used for total wall shear were as follows:

[0151] Expression 1:

$$F_\tau = \frac{32\mu u_o L^2 W}{D_H^2} \text{ (Hagen-Poiseuille)}$$

[0152] Expression 2:

$$F_\tau = \frac{0.4696\mu u_o LW}{\sqrt{2\nu L/u_o}} \text{ (White, } \underline{\text{Viscous Fluid Flow}}\text{, 233-235)}$$

[0153] Time and Spatially-Dependent Film Thickness

[0154] A sketch of the model essentials is given in **FIG. 4**. As indicated, the liquid film thickness, δ, is a function of both axial location, z, and elapsed time since bulk drainage, t. We begin with the continuity equation

$$\frac{\partial\rho}{\partial t} = -(\vec{\nabla}\cdot\rho\vec{v}) \tag{1}$$

[0155] and perform an unsteady mass balance on the film between z and z+Δz: In order to reduce the model to a pseudo 1D approach, we replace the local velocity vector $\vec{v}$ with the cross-sectionally averaged value over the local thickness of the liquid film. Neglecting the density of the surrounding gaseous medium, the local density in the differential volume, ρ, now becomes solely a function of local film thickness and the continuity equation becomes

$$-\frac{1}{\delta}\frac{\partial\delta}{\partial t} = \frac{\partial < v_z >}{\partial z} \tag{2}$$

[0156] where $<v_z>$ is the cross-sectionally averaged velocity at axial location z.

[0157] The following expression gives the velocity distribution for a liquid film as a function of normal distance× from the free (outer) film surface toward the wall surface and inclination angle β relative to the direction of the force of gravity (*Transport Phenomena*, Bird, Stewart, and Lightfoot, 2nd Ed., Wiley and Sons):